Traceability for computationally-intensive metrology
User manual for

## Chebyshev algorithm testing



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## 1. Introduction

PTBs TraCIM system offers testing of Chebyshev minimization algorithms (Minimum-zone method) for the following basic geometric elements:

Line 2D, Circle 2D, Plane, Sphere and Cylinder

The PTB test was provided within the frame of a project named TraCIM, founded by the European Union (EMRP project NEW06 TraCIM - http://www.tracim.eu/) and is under strict quality control of TraCIM e.V. Association.

The test is provided by the PTB TraCIM Online System. For the test a customer will get an XML file with datasets containing points in space for the elements listed above. The points are both randomly and systematically distributed on fragments of these elements mainly representing full features with different random and systematic form deviation components. A few datasets represent partial features.

Task of the customer is to evaluate the parameters of the best-fit geometric elements regarding the Minimum-Zone-criterion and send these back to the TraCIM System. The TraCIM system automatically compares customer results with its own reference values and sends a report with the test result. Evaluation procedures are subject to section 2.

To perform a test it is necessary to provide a client application for communication with the TraCIM Server. In section 3 support for developing a client can be found. The XML data schemata used for data exchange are described in section 4.

Placing an order requires registration at the TraCIM web service (tracim.ptb.de) in order to get access to the web shop for buying tests. After successful purchase of a test a customer will get an order key that allows him to request test data from the TraCIM system with his client application. Registered customers have also access to a public test free of charge but without valid certificate.

In order to check the functionality of a TraCIM client, a charge free test suite including the results for the public test is provided. A registered customer can use this to run the public test from the TraCIM system at any time in order to evaluate the correct function of his client-server communication. See section 3.2 on how to request these test data sets. Additional support will be charged.

### 1.1. Basic data sets

The Chebyshev elements test contains 50 data sets with following basic data set IDs and associated geometric element IDs:

Table 1: Data/Element IDs

| Data set ID | Element ID |
| :--- | :--- |
| C01 - C08 | LINE_2D |
| C09-C16 | CIRCLE_2D |
| C17 - C26 | PLANE |
| C27 - C36 | SPHERE |
| C37 - C50 | CYLINDER |

$>$ Each data set consists of points in 3-dimensional Cartesian space.
$>$ A point is represented by the three coordinates $(\mathrm{x}, \mathrm{y}, \mathrm{z})$.
$>$ For LINE_2D and CIRCLE_2D the z-component of each point is set zero $(x, y, 0)$.
$>$ The amount of points per data set varies between 10 and 631 .
> Each point coordinate value is delivered as decimal number with 20 digits and scientific e-format.
$>$ The values refer to the unit mm (millimeter)
$>$ All point coordinates are within the value range $[-500 \mathrm{~mm},+500 \mathrm{~mm}]$.
$>$ Section 4.1 shows the XML data structure that contains the test data sets.

### 1.2.Result parameters

For each element the following result parameters must be computed by the test customer:
Table 2: Result parameter list

| Element ID | Parameter ID | Description |
| :--- | :--- | :--- |
| LINE_2D | Position | The coordinates of the point on the associated line |
|  | Orientation | The direction cosine values of the line |
|  | Form deviation | The form deviation of the line according to Appendix B, ISO 1101:2008 |
|  | Position | The coordinates of the center point of the associated circle. |
|  | Radius | The radius of the associated circle |
|  | Form deviation | The form deviation of the circle according to Appendix B, ISO <br> $1101: 2008$ |
| SPHERE | Position | The coordinates of the point on the associated plane |
|  | Orientation | The direction cosine giving the normal vector of the associated plane. |
|  | Form deviation | The form deviation of the plane according to Appendix B, ISO <br> $1101: 2008$ |
| CYLINDER | Position | The coordinates of the center point of the associated sphere. |
|  | Radius | The radius of the associated sphere. |
|  | Form deviation | The form deviation of the sphere according to Appendix B, ISO <br> $1101: 2008)$ |
|  | Position | The coordinates of the point on the associated cylinder axis |
| Orientation | The direction cosine values of the associated cylinder axis orientation. |  |
|  | Radius | The radius of the associated cylinder lateral surface. |
|  | Form deviation | The form deviation of the cylinder according to Appendix B, ISO <br> $1101: 2008)$ |

$>$ For all elements the position parameter is a vector in 3-dimensional Cartesian space with components $\mathrm{x}, \mathrm{y}$, and z . The values refer to the unit mm (millimeter) - see 1.1 (Basic Data Sets), remark for 'point coordinates.
> For LINE_2D and CIRCLE_2D elements the z -component value of the position parameter must be zero.
> For LINE_2D, PLANE and CYLINDER elements the orientation parameter is a vector in 3-dimensional Cartesian space with components x, y and z. Each vector must be scaled to length value one (direction cosines).
> For LINE_2D elements the z -component of the Orientation parameter must be zero.
$>$ Radius and form deviation must be positive values. These refer to the unit mm (millimeter).
> All parameter values must be in decimal number format (fixed point or floating point e-format are applicable)
> Section 4.2 shows how to write the test results in a file consistent with the TraCIM system test result data structure.

## 2. Test result evaluation ${ }^{1)}$

During validation the test results of a customer software are compared to the associated reference results in the TraCIM system data base. For this purpose deviations regarding position, orientation, size and form deviation of the geometric elements are evaluated and verified to the specified maximum permissible error values (MPE). The following MPEs are the default values:

| Parameter | Specification of deviation | Default MPE value |
| :--- | :--- | :--- |
| Position | Orthogonal distance of reference <br> position to test result line, plane, axis <br> or center point | $0.1 \mu \mathrm{~m}$ |
| Orientation | Angle between calculated and <br> reference orientation vector | $0.1 \mu \mathrm{rad}$ |
| Radius <br> (size) | Difference between calculated radius <br> and reference value | $0.1 \mu \mathrm{~m}$ |
| Form deviation | Difference between calculated form <br> deviation and reference value | $0.01 \mu \mathrm{~m}$ |

The test passed successfully if all deviations are less than the preset MPE values. After the evaluation the TraCIM system will issue a test report containing a certificate for tests passed or a table listing the results of the comparison for each of the elements where the test failed. The test report is sent back to the customer by the client-server connection.

If the customer selects MPEs other than the default values (only larger values are allowed), then the results will be analyzed with reference to these values. Giving customer MPEs is shown in section 4.3.

1) 'test expiration': When a test was ordered, a 180 day deadline for the customer will start. The customer has to finish the test procedure during this period. 30 days and 3 days before the expiration date, a notification mail will be sent to the customer's email address (as submitted with the registration data).

## 3. Client-server communication

The test procedure is highly automated using internet based data exchange by client-server communication. Thru a client application the user is able to obtain test data and send the
computational results. The TraCIM system server automatically handles requests. The necessary communication between client and server to perform a single test is shown in the following figure:


The data exchanged between client and server is encapsulated within XML messages. A proper specification of the applied XML data schemata is subject to section 4. During processing these messages are treated as plain character strings send between the client application and the TraCIM system server.

### 3.1. Development of a client application

## Configuration of the HTTPS connection:

For communication with the TraCIM server a client application must use an HTTPS (Hypertext Transfer Protocol Secure = encrypted HTTPS) connection that allows to send and receive content in the form of character strings containing messages in XML format. Each HTTPS connection is created from a specific URL (Uniform Resource Locator) regarding requests for test data or requests for test result evaluation.
After opening the connection according to the URL the following configurations must be done:

- Enable output and input operations for the connection
- Set the request method "POST" (request comprising input and output)
- Set connection property "content-Type" to "application/xml"
- Set connection property "accept" to "application/xml"
- Set connection property "content-length" to the amount of characters of the content that is send to the TraCIM system.

Packages for creation and configuration of an HTTPS connection are available for different programming languages such as for (examples):

- Java: java.net API (HttpURLConnection)
- C/C++: Microsoft C++ REST SDK or similar
- C\#: System.Net (.NET 4.8: System.Net.Http) Assembly


## POST request for obtaining test data sets:

The URL of an HTTPS connection for POST request in order to obtain Chebyshev test data sets is
https://tracim.ptb.de/tracim/api/order/<CHEBYSHEV_ORDER_KEY>/test
where <CHEBYSHEV_ORDER_KEY> has to be replaced by the Chebyshev order key purchased at the TraCIM web shop. After creation and configuration of a connection as described above the client application has to send the request message. When the TraCIM system receives the client message it will start creating test data sets and a unique process key. The TraCIM content is returned to the client.

For cases that a customer is not properly registered for the TraCIM System or incorrect order keys the TraCIM server will send an error message.

Client content and TraCIM system content for retrieving test data are presented in section 4.1. Error messages are presented in section 4.4.

## POST request for sending test results and obtaining the certificate:

The URL of an HTTPS connection for a POST request in order to send test results for evaluation and get the certificate for Chebyshev testing is

## https://tracim.ptb.de/tracim/api/test/<PROCESS_KEY>

where <PROCESS_KEY> has to be replaced by the individual process key that was returned by the TraCIM system at the POST request for test data sets.

The client will prepare the XML content (test result parameters, process key, software identification, ....) to be attached to the POST request and will send it to the TraCIM system.

The TraCIM System will receive and evaluate the content and generate a test report. It states whether the test is passed (user results sufficiently accurate) or gives a table with an error outline for all elements 'C01-C50' according to each parameter. In case of a sample test request the test report is not countersigned by PTB seal. The data sent to the client contains the test report encoded in XML.

For the case of an incorrect process key the TraCIM content will be an error message.
Client content and TraCIM system content for retrieving the test evaluation and PDF report are presented in sections 4.2 and 4.3. Error messages are presented in section 4.4.

[^0]
### 3.2. Public sample data for client testing

Sample data denotes a complete Chebyshev test for backtracking errors within the client application that could put a successful commercial test run on the risk. For any registered customer the data is free of charge with unlimited request amount. In comparison to a test with commercial test data the certificate returned by the server is not countersigned by PTB as legally valid certificate.

The sample order key is available for registered customers. Please check the website (https://tracim.ptb.de ) or send a mail to info.tracim@ptb.de for further information.

## 4. XML message content

### 4.1. Test data

Test data returned by the TraCIM system composed according to an XML scheme with three major elements for order identification, process identification and the test data sets. The order element contains a copy of the order key, a date of the creation of the test data and a date for the expiration of the test data (deadline for sending test results for evaluation). The process element contains the process key associated with the test data request. The test element contains several point clouds with vector elements that give the $\mathrm{x}, \mathrm{y}$ and z coordinates of the test data. For each point cloud the XML tag "basicID" contains the unique data set id (C01-C50) and the XML tag "computationalObject" contains the corresponding geometrical element (LINE_2D, CIRCLE_2D, PLANE, SPHERE, CYLINDER).

```
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<tracim:tracim
            xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
            xmlns:chebyshev="http://tracim.ptb.de/chebyshev/test"
            xmlns:tracim="http://tracim.ptb.de/tracim">
<tracim:order>
    <tracim:key>[ORDER_KEY]</tracim:key>
</tracim:order>
<tracim:process>
    <tracim:key>[PROCESS_KEY]</tracim:key>
</tracim:process>
<tracim: tests xsi:type="chebyshev:chebyshevTestPackage">
    <chebyshev:package>
        <chebyshev:testElement>
            <chebyshev:basicID>C01</chebyshev:basicID>
            <chebyshev:computationObject>LINE_2D</chebyshev:computationObject>
            <chebyshev:pointCloud>
                <chebyshev:vectors>
                    <chebyshev:x>-44.7893023434000000</chebyshev:x>
                    <chebyshev:y>58.34322222322222200</chebyshev:y>
                        <chebyshev:z>0.000000000000000000</chebyshev:z>
                </chebyshev:vectors>
                <chebyshev:vectors>
                    <chebyshev:x>-44.7893023434000000</chebyshev:x>
                    <chebyshev:y>58.34322222322222200</chebyshev:y>
                    <chebyshev:z>0.000000000000000000</chebyshev:z>
                </chebyshev:vectors>
```

[..]
</chebyshev:pointCloud> </chebyshev:testElement>
[chebyshev:testElement](chebyshev:testElement)
[chebyshev:basicID](chebyshev:basicID)C37</chebyshev:basicID>
[chebyshev:computationObject](chebyshev:computationObject)CYLINDER</chebyshev:computationObject>
[...]
</chebyshev:testElement>
[...]
</chebyshev:package>
</tracim:tests>
</tracim:tracim>

### 4.2.Test results

For sending the calculated results of software under test to the TraCIM system the XML scheme below must be used. The client has to fill in the following information (next page):

- [PROCESS_KEY] key received in message from section 4.1
- [CUSTOMER_NAME]
- [SOFTWARE_VENDOR]
- [SOFTWARE_NAME]
- [SOFTWARE_VERSION]
- [SOFTWARE_REVISION] customer name from TraCIM system registration name of the manufacturer of the software (might be the same as customer name)
name of software that computed the test results version of software that computed the test results revision of software that computed the test results (optional)

Further the client has to specify the MPE values for position, orientation, size and form deviation that should be used for test evaluation. User MPE values are subjected to unit millimeter for length and rad for orientation.

This information is followed by elements that contain the result parameters of the software under test. For each test data set the client has to specify a single results element set containing the basic data set id (C01 - C50) and the geometrical element id (LINE_2D, CIRCLE_2D, PLANE, SPHERE, CYLINDER).

```
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<chebyshev:chebyshevResultPackage
    xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
    xmlns:chebyshev="http://tracim.ptb.de/chebyshev/test"
    xmlns:tracim="http://tracim.ptb.de/tracim">
<chebyshev:processKey>[PROCESS_KEY]</chebyshev:processKey>
<chebyshev:customer>[CUSTOMER_NAME]</chebyshev:customer>
<chebyshev:softwareName>[SOFTWARE_NAME]</chebyshev:softwareName>
<chebyshev:softwareVersion>[SOFTWARE_VERSION]</chebyshev:softwareVersion>
<chebyshev:softwareRev>[SOFTWARE_REVISION]</chebyshev:softwareRev>
<chebyshev:mpe_position>0.0001</chebyshev:mpe_position>
<chebyshev:mpe_orientation>0.0000001</chebyshev:mpe_orientation>
<chebyshev:mpe_size>0.0001</chebyshev:mpe_size>
<chebyshev:mpe_formDeviation>0.00001</chebyshev:mpe_formDeviation>
<chebyshev:resultPackage>
    <chebyshev:results>
```

```
            <chebyshev:basicID>C01</chebyshev:basicID>
            <chebyshev:computationObject>LINE_2D</chebyshev:computationObject>
            <chebyshev:refParameter>
                    <chebyshev:positionX>0.000001</chebyshev:positionX>
                    <chebyshev:positionY>0.000005</chebyshev:positionY>
                    <chebyshev:positionZ>0.000000</chebyshev:positionZ>
                    <chebyshev:orientationX>1.00000000021</chebyshev:orientationX>
                    <chebyshev:orientationY>-0.00000000005</chebyshev:orientationY>
                    <chebyshev:orientationZ>0.0000000000</chebyshev:orientationZ>
                    <chebyshev:formDeviation>0.0003578</chebyshev:formDeviation>
            </chebyshev:refParameter>
        </chebyshev:results>
        <chebyshev:results>
            <chebyshev:basicID>C37</chebyshev:basicID>
            <chebyshev:computationObject>CYLINDER</chebyshev:computationObject>
            [...]
        </chebyshev:results>
            [...]
    </chebyshev:resultPackage>
    </chebyshev:chebyshevResultPackage>
```

According to the result parameter list in section 1.2. a client has to provide the following types of parameter elements:

- positionX $x$-coordinate of the position parameter
- positionY $y$-coordinate of the position parameter
- positionZ $z$ - coordinate of the position parameter
- orientationX $x$ - component of the orientation vector
- orientationY $y$ - component of the orientation vector
- orientationZ $z$ - component of the orientation vector
- radius radius for 'CIRCLE_2D', 'SPHERE' and 'CYLINDER'
- formDeviation form deviation associated with the Chebyshev element

Entries for parameters that are not part of the geometrical element parameter specification have not to be written in the scheme (in the example above this applies for the radius parameter in the context of LINE_2D result parameters). Exceptions are the z-coordinate and z-component of position and orientation for the two-dimensional elements LINE_2D and CIRCLE_2D. The client must fill in these with value zero. All values that are entered into the scheme must apply with the requirements in section 1.2. In addition these values have to be character strings compatible with UTF-8 format.

### 4.3. Test report

After validation the TraCIM system returns XML data according to a XML scheme with the validation result. It comprises the three elements

- Passed
- Report
- reportPDF
true, if the software passed the test successful, else false character String with short report on the test evaluation character String with the test report PDF

In order to create the test report PDF document as PDF file, the "reportPDF" character string can be converted and written to new file with the proper ".pdf" file name extension.

```
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<tracim:tracim
xmlns:xsi='hhttp://www.w3.org/2001/XMLSchema-instance">
xmlns:chebyshev="http://tracim.ptb.de/chebyshev/test"
xmlns:tracim="http://tracim.ptb.de/tracim">
    <tracim:validation>
    <tracim:passed>true</tracim:passed>
    <tracim:report></tracim:report>
    <tracim:reportPDF>1)JKD5iuDUD098IHh[...]</tracim:reportPDF>
</tracim:validation>
</tracim:tracim> 1) 'base64' encoded
```


### 4.4. TraCIM Error Messages

The following XML code is sent by the server in case of errors, where [error code] and [error description] are replaced with values according to table 4 (after the XML excerpt).

```
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<tracim:tracim
    xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
    xmlns:chebyshev="http://tracim.ptb.de/chebyshev/test"
    xmlns:tracim="http://tracim.ptb.de/tracim">
    <tracim:error>
        <tracim:code>[error code]</tracim:code>
        <tracim:description>[error description]</tracim:description>
    </tracim:error>
</tracim:tracim>
```

Table 4 : Error messages ${ }^{1)}$

| [error code] | [error description] |
| :--- | :--- |
| 403 | Customer not active. |
| 403 | No more tests left for this order. |
| 403 | Order is not active. |
| 403 | Order already expired. |
| 403 | Test process already closed. |
| 404 | Expert Extension with ID " < ExtensionId>" does not exist. |
| 404 | Unknown order key. |
| 404 | Unknown test process key. |
| 400 | Process key does not match! |
| 400 | Customer element is blank! |
| 400 | Software vendor element is blank! |
| 400 | Software name element is blank! |
| 400 | Software version element is blank! |
| 400 | MPE elements are not present. You are allowed to set MPEs to zero but the elements <br> have to be present! |
| 400 | Check your MPEs! If you set one MPE you have to set them all. |
| 400 | Check your MPEs! Negative MPEs are not allowed. |
| 400 | The MPEs are smaller than the smallest permitted value. Please consult the manual. |


| 400 | Your list contains less or more than 50 computational objects! |
| :--- | :--- |
| $400^{\prime}$ | Parameter set for at least one computational object (<ComputationObject>) is not <br> complete! |
| $400^{\prime}$ | Check the names of the computational objects! Expected are CYLINDER, CIRCLE_2D, <br> PLANE, LINE_2D, SPHERE (all case sensitive). |
| $\mathbf{4 0 0}$ | Error processing the XML file. |
| 500 | Unknown reason |

[^1]
### 4.5.TraCIM schemata

The XML schemata can be obtained from the following URL's:
https://tracim.ptb.de/tracim/api/schema/PTBWHZ_MATH_CHEBYSHEV_v1_test.xsd ${ }^{1)}$ https://tracim.ptb.de/tracim/api/schema/PTBWHZ_MATH_CHEBYSHEV_v1_result.xsd ${ }^{2)}$ https://tracim.ptb.de/tracim/api/schema/tracim.xsd ${ }^{3)}$

## Notes:

1) Test data schema; 2) Result data schema; 3) Report schema

It is strongly recommended, that a customer in order to test client functions uses the schemata during client development.

## 5. The test report

The report can be extracted from the string stream (4.3, XML tag 'tracim: reportPDF' - 'base64' encoded) into a PDF file ready for printing. The layout follows the rules for test reports as used in the PTB (official header, legal statements etc.)

## 6. How to get support

- technical support for client software : info.tracim@ptb.de
- support regarding computational aims : info.tracim@ptb.de
- Provider support (contact, fees, ...) : info.tracim@ptb.de


## 7. Legal and copyright information

Copyright for this document: PTB. The document content is subject to change without notice.
8. Appendices

A - Reference parameter values for example test data sets
B - Underlying computational aims
C - Design of the Chebyshev test point clouds

## Appendix A - Reference parameter values for example test data sets

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Tables ' $a$ ' to 'e' are showing reference parameter values for the public test data sets of elements Line 2D, Circle 2D, Plane, Sphere and Cylinder. These are decimal numbers up to 10 digits precision. Values for position, radius and form deviation are subjected to the dimension millimeter. The components of Orientation are values in length units (no physical dimension).
a) Line 2 D

| Basic ID | Position | Orientation | Form deviation | Number of points |
| :---: | :---: | :---: | :---: | :---: |
| C01 | $\begin{aligned} & x:-10.0 \\ & y:-100.0 \end{aligned}$ | $\begin{aligned} & x: 1.0 \\ & y: 0.0 \end{aligned}$ | 0.022 | 260 |
| C02 | $\begin{aligned} & x:-240.0 \\ & y: 50.0 \end{aligned}$ | $\begin{aligned} & x: 0.0 \\ & y: 1.0 \end{aligned}$ | 0.02 | 631 |
| C03 | $\begin{aligned} & x: 410.0 \\ & y: 405.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}: 0.7071067811865 \\ & \mathrm{y}: 0.7071067811865 \end{aligned}$ | 0.001 | 120 |
| C04 | $\begin{aligned} & x: 130.0 \\ & y: 5.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}:-0.7071067811865 \\ & \mathrm{y}: 0.7071067811865 \end{aligned}$ | 0.005 | 50 |
| C05 | $\begin{aligned} & x:-95.0 \\ & y:-103.0 \end{aligned}$ | $\begin{aligned} & x: 0.8944271909999 \\ & y: 0.4472135954999 \end{aligned}$ | 0.002 | 10 |
| C06 | $\begin{aligned} & x: 5.0 \\ & y:-340.0 \end{aligned}$ | x: 0.4472135954999 <br> y: 0.8944271909999 | 0.015 | 161 |
| C07 | $\begin{aligned} & x: 300.0 \\ & y:-250.0 \end{aligned}$ | $\begin{aligned} & x:-0.0995037190209 \\ & y: 0.9950371902099 \end{aligned}$ | 0.05 | 80 |
| C08 | $\begin{aligned} & \mathrm{x}:-125.0 \\ & y: 345.0 \end{aligned}$ | $\begin{aligned} & \text { x: -0.9977851578566 } \\ & y: 0.0665190105237 \end{aligned}$ | 0.01 | 331 |

b) Circle 2D

| Basic ID | Position | Radius | Form deviation | Number of points |
| :---: | :---: | :---: | :---: | :---: |
| C09 | $\begin{aligned} & x: 400.0 \\ & y: 390.0 \end{aligned}$ | 20.0 | 0.05 | 250 |
| C10 | $\begin{aligned} & x:-200.0 \\ & y: 180.0 \end{aligned}$ | 200.0 | 0.022 | 120 |
| C11 | $\begin{aligned} & x:-250.0 \\ & y:-250.0 \end{aligned}$ | 125.0 | 0.005 | 260 |
| C12 | $\begin{aligned} & x: 105.0 \\ & y:-57.0 \end{aligned}$ | 250.0 | 0.02 | 250 |
| C13 | $\begin{aligned} & x: 0.0 \\ & y:-305.0 \end{aligned}$ | 50.0 | 0.005 | 78 |
| C14 | $\begin{aligned} & x: 280.0 \\ & y:-300.0 \end{aligned}$ | 12.0 | 0.002 | 216 |
| C15 | $\begin{aligned} & x: 100.0 \\ & y: 120.0 \end{aligned}$ | 25.0 | 0.001 | 30 |
| C16 | $\begin{aligned} & \mathrm{x}: 150.0 \\ & \mathrm{y}: 50.0 \end{aligned}$ | 65.0 | 0.01 | 450 |

c) Plane

| Basic ID | Position | Orientation | Form deviation | Number of points |
| :---: | :---: | :---: | :---: | :---: |
| C17 | $\begin{aligned} & \mathrm{x}: 0.0 \\ & \mathrm{y}: 0.0 \\ & z: 0.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}: 0.0 \\ & \mathrm{y}: 1.0 \\ & z: 0.0 \end{aligned}$ | 0.075 | 50 |
| C18 | $\begin{aligned} & x: 400.0 \\ & y: 400.0 \\ & z: 400.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}: 1.0 \\ & \mathrm{y}: 0.0 \\ & \mathrm{z}: 0.0 \end{aligned}$ | 0.001 | 145 |
| C19 | $\begin{aligned} & x: 350.0 \\ & y: 0.0 \\ & z:-250.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}: 0.0 \\ & \mathrm{y}: 0.0 \\ & \mathrm{z}: 1.0 \end{aligned}$ | 0.01 | 397 |
| C20 | $\begin{aligned} & x: 0.0 \\ & y:-270.0 \\ & z:-280.0 \end{aligned}$ | $\begin{aligned} & x: 0.0 \\ & y: 0.7071067811865 \\ & z: 0.7071067811865 \end{aligned}$ | 0.005 | 136 |
| C21 | $\begin{aligned} & \mathrm{x}: 310.0 \\ & \mathrm{y}:-95.0 \\ & \mathrm{z}: 210.0 \end{aligned}$ | $\begin{aligned} & x: 0.7071067811865 \\ & y: 0.0 \\ & z: 0.7071067811865 \end{aligned}$ | 0.003 | 10 |
| C22 | $\begin{aligned} & x: 185.0 \\ & y: 155.0 \\ & z:-10.0 \end{aligned}$ | $\begin{aligned} & x: 0.7071067811865 \\ & y: 0.7071067811865 \\ & z: 0.0 \end{aligned}$ | 0.005 | 20 |
| C23 | $\begin{aligned} & \text { x: } 110.0 \\ & \text { y: } 10.0 \\ & \text { z: -310.0 } \end{aligned}$ | $\begin{aligned} & x: 0.5934424260562 \\ & y: 0.6593804733957 \\ & z: 0.46156633137705 \end{aligned}$ | 0.022 | 174 |
| C24 | $\begin{aligned} & x: 20.0 \\ & y:-15.0 \\ & z: 20.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}:-0.6388765649999 \\ & \mathrm{y}: 0.5111012519999 \\ & z: 0.5749889084999 \end{aligned}$ | 0.05 | 450 |
| C25 | $\begin{aligned} & x:-120.0 \\ & y: 130.0 \\ & z: 170.0 \end{aligned}$ | $\begin{aligned} & x: 0.5232045649263 \\ & y:-0.5232045649263 \\ & z: 0.6726915834767 \end{aligned}$ | 0.09 | 451 |
| C26 | $\begin{aligned} & \mathrm{x}:-50.0 \\ & \mathrm{y}:-60.0 \\ & z:-50.0 \end{aligned}$ | $\begin{aligned} & \text { x: }-0.5533715710928 \\ & y:-0.6225430174794 \\ & z: 0.5533715710928 \end{aligned}$ | 0.025 | 35 |

d) Sphere

| Basic ID | Position | Radius | Form deviation | Number of points |
| :---: | :---: | :---: | :---: | :---: |
| C27 | $\begin{aligned} & x:-280.0 \\ & y:-300.0 \\ & z:-220.0 \end{aligned}$ | 70.0 | 0.015 | 144 |
| C28 | $\begin{aligned} & x:-250.0 \\ & y: 240.0 \\ & z: 250.0 \end{aligned}$ | 130.0 | 0.02 | 350 |
| C29 | $\begin{aligned} & x: 150.0 \\ & y:-120.0 \\ & z:-90.0 \end{aligned}$ | 25.0 | 0.005 | 25 |
| C30 | $\begin{aligned} & x: 150.0 \\ & y: 150.0 \\ & z:-110.0 \end{aligned}$ | 210.0 | 0.04 | 182 |
| C31 | $\begin{aligned} & x: 100.0 \\ & y: 350.0 \\ & z: 0.0 \end{aligned}$ | 15.0 | 0.003 | 50 |
| C32 | $\begin{aligned} & x: 300.0 \\ & y: 0.0 \\ & z: 280.0 \end{aligned}$ | 20.0 | 0.02 | 241 |
| C33 | $\begin{aligned} & x:-90.0 \\ & y:-90.0 \\ & z: 0.0 \end{aligned}$ | 55.0 | 0.01 | 55 |
| C34 | $\begin{aligned} & x: 0.0 \\ & y: 0.0 \\ & z: 0.0 \end{aligned}$ | 5.0 | 0.001 | 120 |
| C35 | $\begin{aligned} & x: 0.0 \\ & y=0.0 \\ & z: 0.0 \end{aligned}$ | 260.0 | 0.05 | 120 |
| C36 | $\begin{aligned} & x: 280.0 \\ & y:-200.0 \\ & z: 300.0 \end{aligned}$ | 60.0 | 0.01 | 210 |

e) Cylinder

| Basic ID | Position | Orientation | Radius | Form deviation | Number of points |
| :---: | :---: | :---: | :---: | :---: | :---: |
| C37 | $\begin{aligned} & x: 0.0 \\ & y: 0.0 \\ & z: 0.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}: 1.0 \\ & \mathrm{y}: 0.0 \\ & \mathrm{z}: 0.0 \end{aligned}$ | 250.0 | 0.025 | 250 |
| C38 | $\begin{aligned} & x: 230.0 \\ & y: 310.0 \\ & z: 260.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}: 0.0 \\ & \mathrm{y}: 1.0 \\ & \mathrm{z}: 0.0 \end{aligned}$ | 50.0 | 0.005 | 75 |
| C39 | $\begin{aligned} & \mathrm{x}:-250.0 \\ & \mathrm{y}:-250.0 \\ & \mathrm{z}:-250.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}: 0.0 \\ & \mathrm{y}: 0.0 \\ & \mathrm{z}: 1.0 \end{aligned}$ | 12.5 | 0.002 | 216 |
| C40 | $\begin{aligned} & x: 90.0 \\ & y:-110.0 \\ & z:-80.0 \end{aligned}$ | $\begin{aligned} & x: 0.7071067811865 \\ & y: 0.7071067811865 \\ & z: 0.0 \end{aligned}$ | 20.0 | 0.01 | 250 |
| C41 | $\begin{aligned} & x:-90.0 \\ & y:-200.0 \\ & z: 100.0 \end{aligned}$ | $\begin{aligned} & x: 0.0 \\ & y: 0.7071067811865 \\ & z: 0.7071067811865 \end{aligned}$ | 200.0 | 0.02 | 112 |
| C42 | $\begin{aligned} & x: 240.0 \\ & y: 280.0 \\ & z:-235.0 \end{aligned}$ | $\begin{aligned} & x: 0.7071067811865 \\ & y: 0.0 \\ & z: 0.7071067811865 \end{aligned}$ | 37.5 | 0.005 | 264 |
| C43 | $\begin{aligned} & \mathrm{x}: 60.0 \\ & \mathrm{y}: 50.0 \\ & \mathrm{z}: 80.0 \end{aligned}$ | $\begin{aligned} & x: 0.8479983040051 \\ & y: 0.3179993640019 \\ & z: 0.4239991520025 \end{aligned}$ | 125.0 | 0.05 | 480 |
| C44 | $\begin{aligned} & x: 0.0 \\ & y: 0.0 \\ & z: 0.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}:-0.5391638660171 \\ & \mathrm{y}: 0.5391638660171 \\ & \mathrm{z}: 0.6469966392206 \end{aligned}$ | 92.0 | 0.015 | 125 |
| C45 | $\begin{aligned} & x:-20.0 \\ & y:-20.0 \\ & z: 20.0 \end{aligned}$ | $\begin{aligned} & x:-0.3810003810005 \\ & y:-0.8890008890013 \\ & z: 0.2540002540003 \end{aligned}$ | 65.0 | 0.015 | 455 |
| C46 | $\begin{aligned} & x: 300.0 \\ & y:-415.0 \\ & z: 320.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}: 0.2407717061715 \\ & \mathrm{y}:-0.4815434123431 \\ & \mathrm{z}: 0.8427009716003 \end{aligned}$ | 25.0 | 0.001 | 30 |

Next page: Cylinder values continued

Cylinder values continued from previous page

| Basic ID | Position | Orientation | Radius | Form deviation | Number of points |
| :---: | :---: | :---: | :---: | :---: | :---: |
| C47 | $\begin{aligned} & \mathrm{x}: 55.0 \\ & \mathrm{y}: 50.0 \\ & \mathrm{z}: 50.0 \end{aligned}$ | $\begin{aligned} & x: 0.1230914909793 \\ & y: 0.9847319278346 \\ & z: 0.1230914909793 \end{aligned}$ | 3.25 | 0.005 | 40 |
| C48 | $\begin{aligned} & \mathrm{x}: 50.0 \\ & \mathrm{y}: 45.0 \\ & \mathrm{z}:-45.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}: 0.9630868246861 \\ & \mathrm{y}:-0.1203858530857 \\ & \mathrm{z}: 0.2407717061715 \end{aligned}$ | 4.35 | 0.008 | 66 |
| C49 | $\begin{aligned} & x:-55.0 \\ & y: 50.0 \\ & z: 50.0 \end{aligned}$ | $\begin{aligned} & \mathrm{x}:-0.1162476387438 \\ & \mathrm{y}: 0.9299811099505 \\ & \mathrm{z}: 0.3487429162314 \end{aligned}$ | 5.375 | 0.005 | 50 |
| C50 | $\begin{aligned} & \mathrm{x}: 50.0 \\ & \mathrm{y}:-50.0 \\ & \mathrm{z}: 50.0 \end{aligned}$ | $\begin{aligned} & \text { x: -0.9630868246861 } \\ & \text { y: -0.1203858530857 } \\ & \text { z: } 0.2407717061715 \end{aligned}$ | 1.65 | 0.002 | 30 |

## Appendix B - Underlying computational aims

Version 2014-10-24

The input data for the Minimum-Zone element fitting is a so called point cloud

$$
P:=\left\{P_{1}, \ldots, P_{n}\right\}
$$

of $n \in \mathbb{N}$ point coordinates $P_{i} \in \mathbb{R}^{k}, k=2,3$ for all indices $i$ from the index set

$$
I:=\{1, \ldots, n\} .
$$

The point cloud represents a discrete sample of surficial measurement points from a workpiece that has the shape of a basic geometric element lateral surface in the 3D-case or respectively a lateral curvature in the 2D-case. The elements subject to this appendix are Line 2D, Circle 2D, Plane, Sphere and Cylinder. By

$$
b \in G \subset \mathbb{R}^{m}
$$

with $m \in \mathbb{N}, m \geq 1$ and $G \neq \emptyset$ we introduce a parameter vector that defines the upper elements in their ideal geometrical shape. The dimension $m$ and range of valid parameters $G$ have individual definitions for each geometric element. Then for a fixed parameter $b$ each of the ideal shaped geometric elements is implicitly defined by

$$
A(b):=\left\{v \in \mathbb{R}^{k} \mid f(v, b)=0\right\}
$$

where $f: \mathbb{R}^{k} \times \mathbb{R}^{m} \rightarrow \mathbb{R}$ denotes a function that gives the orthogonal distance between an arbitrary point in $\mathbb{R}^{k}$ and the ideal surface or curvature of the geometric element.

In general there may not be a parameter vector $b$ that satisfies $P \subseteq A(b)$ due to deviations in form and size of the workpiece in comparison with the underlying ideal geometric element. Hence applications in geometrical dimensioning and tolerancing require additional fitting criteria to provide definitions of associated geometrical elements for the point clouds.

One of these criteria is the Minimum-Zone best-fit. Its objective is

$$
\begin{equation*}
\min _{b \in G} \max _{i \in I}\left|f_{i}(b)\right| \tag{1}
\end{equation*}
$$

and

$$
f_{i}(b):=f\left(P_{i}, b\right)
$$

give the orthogonal distance of the points from the point cloud $P$ to the geometric element with parameters $b$. Problem (1) is referred to as a discreet minimax-problem in optimization theory. Moreover the objective

$$
F(b):=\max _{i \in I}\left|f_{i}(b)\right|
$$

is the norm of all orthogonal distance values in the maximum norm on the real value vector space $\mathbb{R}^{n}$ that is commonly known as Chebyshev-norm and hence (1) is called Chebyshev-fitting, too. The table below gives formal mathematical definitions for parameter vectors, parameter space and orthogonal distance functions.

Table 1: Geometry parameter and distance function definitions for Minimum-Zone elements Line 2D, Circle 2D, Plane, Sphere and Cylinder.

| Element | Parameter vectors b | Range of parameters $G$ | Distance function |
| :---: | :---: | :---: | :---: |
| Circle 2D | $b:=\left(c_{x}, c_{y}, r\right)^{T}$ | $\mathbb{R}^{2} \times \mathbb{R}_{+}$ | $f(Q, b):=\\|Q-C\\|-r$ |
|  | Center $C=\left(c_{x}, c_{y}\right)^{T}$ | $\mathbb{R}^{2}$ |  |
|  | Radius $r$ | $\mathbb{R}_{+}$ |  |
| Line 2D | $b:=\left(c_{x}, c_{y}, v_{x}, v_{y}\right)^{T}$ | $\mathbb{R}^{2} \times S_{1}(\mathbf{0})$ | $\begin{aligned} & f(Q, b):=\langle Q-C, n\rangle \quad \text { with } \\ & n:=\left(-v_{y}, v_{x}\right)^{T} \end{aligned}$ |
|  | Orthogonal projection of point data set barycenter on line $C=$ $\left(c_{x}, c_{y}\right)^{T}$ | $\mathbb{R}^{2}$ |  |
|  | Line direction $v=\left(v_{x}, v_{y}\right)^{T}$ | $S_{1}(\mathbf{0}) \subset \mathbb{R}^{2}$ |  |
| Plane | $b:=\left(c_{x}, c_{y}, c_{z}, n_{x}, n_{y}, n_{z}\right)^{T}$ | $\mathbb{R}^{3} \times S_{1}(\mathbf{0})$ | $f(Q, b):=\langle Q-C, n\rangle$ |
|  | Orthogonal projection of point data set barycenter on plane $C=$ $\left(c_{x}, c_{y}, c_{z}\right)^{T}$ | $\mathbb{R}^{3}$ |  |
|  | Plane normal $n=\left(n_{x}, n_{y}, n_{z}\right)^{T}$ | $S_{1}(\mathbf{0}) \subset \mathbb{R}^{3}$ |  |
| Sphere | $b:=\left(c_{x}, c_{y}, c_{z}, r\right)^{T}$ | $\mathbb{R}^{3} \times \mathbb{R}_{+}$ | $f(Q, b):=\\|Q-C\\|-r$ |
|  | Center point $C=\left(c_{x}, c_{y}, c_{z}\right)^{T}$ | $\mathbb{R}^{3}$ |  |
|  | Radius $r$ | $\mathbb{R}_{+}$ |  |
| Cylinder | $b:=\left(c_{x}, c_{y}, c_{z}, v_{x}, v_{y}, v_{z}, r\right)^{T}$ | $\begin{aligned} & \mathbb{R}^{3} \times S_{1}(\mathbf{0}) \\ & \times \mathbb{R}_{+} \\ & \hline \end{aligned}$ | $f(Q, b):=\\|(Q-C) \times v\\|-r$ |
|  | Orthogonal projection of point data set barycenter on cylinder axis $C=\left(c_{x}, c_{y}, c_{z}\right)^{T}$ | $\mathbb{R}^{3}$ |  |
|  | Axis direction $v=\left(v_{x}, v_{y}, v_{z}\right)^{T}$ | $S_{1}(\mathbf{0}) \subset \mathbb{R}^{3}$ |  |
|  | Radius $r$ | $\mathbb{R}_{+}$ |  |

## Computation of projected centroids:

The centroid of the point cloud (point data set barycenter) is

$$
\bar{P}:=\frac{1}{n} \sum_{i=1}^{n} P_{i} .
$$

Then table 2 gives the evaluation of the projected centroid for several elements.

| Element | Computation projected centroid $C$ |
| :--- | :--- |
| Line 2D, Cylinder | $C$ solves $\langle\bar{P}-C, v\rangle=0$ |


| Plane | C solves $(\bar{P}-C) \times n=0$ |
| :--- | :--- |

Table 2: Rules for projected centroid computation

## Computation of element form deviation:

For each geometrical element the form deviation is defined by

$$
e f:=2 F(b)
$$

For parameters $b$ from the Minimum-Zone best-fit (1) this yields form deviation assessment according to requirements from ISO 1101 - Appendix B [4]. The form deviation describes the width of an area enclosing all points in the point cloud - the minimum-zone. The following geometrical depictions apply for Chebyshev elements:

- Line 2D: The Minimum-Zone is bounded by two geometrical ideal straight lines with minimum distance that are parallel to the Chebyshev line. The orthogonal distance of the bounding lines is the associated form deviation. The Chebyshev line is the median line between the bounding lines.
- Circle 2D: The Minimum-Zone is bounded by two geometrical ideal concentric circles with center point coincident with the center point of the Chebyshev circle and minimum radius distance. The radius distance is the associated form deviation. The Chebyshev circle is the median circle between the bounding circles.
- Plane: The Minimum-Zone is bounded by two geometrical ideal planes with minimum distance that are parallel to the Chebyshev plane. The orthogonal distance of the bounding planes is the form deviation. The Chebyshev plane is the median plane between the bounding planes.
- Sphere: The Minimum-Zone is bounded by two geometrical ideal concentric spheres with center point coincident with the center point of the Chebyshev sphere and minimum radius distance. The radius distance is the associated form deviation. The Chebyshev sphere is the median sphere between the bounding spheres.
- Cylinder: The Minimum-Zone is bounded by two geometrical ideal cylinders coaxial to the Chebyshev cylinder and with minimum radius distance. The form deviation is the radius distance is the associated form deviation. The Chebyshev cylinder is the median cylinder between the bounding cylinders.


## Remarks on properties of the Chebyshev element fitting:

a) The parameter selection in Table 1 is subjected to normative specifications for geometrical element algorithm testing in ISO 10360 part 6 [1]. For computational evaluation of the elements with metrological software different parameterization is possible.
b) Depending on the structure of the point cloud problem (1) could have a unique global solution, several equivalent global solutions or even no solution. For example symmetric data sets may contain equivalent global solutions. Moreover for collinear points the fitting of Circle 2D has an invalid solution with finite radius.
c) Despite the global solution properties a point cloud could contain additional local solution that are consistent with mathematical solution properties for problem (1) but have a greater value for form deviation than a global solution. Hence the application of numerical methods for the element computation requires the application of additional global search strategies in order to find a global solution instead of a local solution.
d) For solving problem (1) it is recommended to use numerical methods that are able to

## Further references for details:

[1] DIN EN ISO 10360-6: Geometrical Product Specifications (GPS) - Acceptance and reverification tests for coordinate measuring machines (CMM) - Part 6: Estimation of errors in computing Gaussian associated features (ISO 10360-6:2001+Cor. 1:2007), German version EN ISO 10360-6:2001+AC:2007
[2] G.T. Anthony, B. Bittner, R. Drieschner, J. Kok, et. al.: Chebyshev reference software for the evaluation of coordinate measuring machine data, Commission of the European Communities, Programme for Applied Metrology and Chemical Analysis (BCR), Report EUR 15304 EN, October 1993
[3] R. Drieschner: Chebyshev approximation to data by geometric elements, Numerical Algorithms 5, pp 509-522, 1993
[4] DIN EN ISO 1101: Geometrical Product Specification (GPS) - Geometrical tolerancing Tolerances of form, orientation, location and run-out (ISO 1101:2004); German version EN ISO 1101:2005; August 2008

## Appendix C - Design of the Chebyshev test point clouds

Version 2016-02-01

## a) Test data sets for 2D line elements

Point clouds for 2D line elements represent coherent line segments in 2D Cartesian space. The amount of points in the PTB's different test data sets varies between 10 and 631 . Some of the data sets have equidistant points. The list of point coordinates in the XML files are ordered in this case. The remaining 2D line data sets have points randomly placed along the line segment. Here the point coordinate data is not in a specific order in the XML files. Further all points differ from an ideal straight line by random and systematic form deviation components. Form types 'random only', 'random and convex' and 'random and one harmonic' are implemented within the test data sets. Figures 1.1 to 1.3 give examples for point positioning and possible shape of data sets. In the figures the grey area marks the zone of minimum width which is the smallest possible form deviation of the associated minimum-zone line. Green, red and yellow dots mark measuring points. The red and yellow points have the maximum orthogonal distance from the associated line. For better visualisation the form deviation in the examples is much higher than in the actual TraCIM test data sets.


Figure 1.1: Example of 2D line segment with form deviations of type 'random only'. The upper picture shows a line segment with randomly generated from deviation and points randomly placed along the line. The lower picture shows a line segment with equally spaced points across the line and with randomly generated form deviations.


Figure 1.2: Example of 2D line segment with form deviations of type 'random and convex'. Both pictures show lines with equally spaced points across the line, but with different convex shapes.


Figure 1.3: Example of 2D line segment with form deviations of type 'random and one harmonic'. Both pictures show lines with equally spaced points along the line segment, but with different frequencies of the harmonic shape.

Details of public 2D line test data are listed below. These also apply to validation data sets.

## 2D Line CO1

Number of points: 260
One systematic convex and one random form deviation component, equidistant points, data represent one coherent line segment

## 2D Line CO2

Number of points: 631
One systematic harmonic and one random form deviation component, equidistant points, data represent one coherent line segment

## 2D Line CO3

Number of points: 120
One random form deviation component, equidistant points, data represent one coherent line segment

## 2D Line CO4

Number of points: 50
One random form deviation component, equidistant points, data represent one coherent line segment

## 2D Line C05

Number of points: 10
One random form deviation component, random space between points, data represent one coherent line segment

## 2D Line C06

Number of points: 161
One systematic harmonic and one random form deviation component, equidistant points, data represent one coherent line segment

## 2D Line C07

Number of points: 80
One systematic convex and one random form deviation component, randomly spaced points, data represent one coherent line segment

## 2D Line C08

Number of points: 331
One systematic harmonic and one random form deviation component, equidistant points, data represent one coherent line segment
b) Test data sets for 2D circle elements

Point clouds for 2D circle elements represent full circles in 2D Cartesian space. The number of points in different test data sets varies between 30 and 450 . Some of the data sets have equidistant points. The list of point coordinates in the XML files are ordered in this case. The remaining 2D circle data sets have randomly spaced points across the circle. Here the points are not listed in a specific order in the XML files. Further all points differ from an ideal circle shape by random and systematic form deviation. Form types 'random only', 'random and one harmonic' and 'random and two harmonics' are implemented and are represented in different the test data set. Figures 2.1 to 2.3 give examples for the point positioning and possible shapes of data sets. In the figures the three concentric circles mark the minimum-zone width which is the smallest possible form deviation of the associated circle. The minimum-zone circle is the median circle between the outer and inner circle. Green, red and yellow dots mark measuring points. The red and yellow points have the maximum orthogonal distance from the associated median circle. For better visualisation the form deviation in the examples is much higher than in the actual TraCIM test data sets.


Figure 2.1: Example of 2D minimum-zone circle with randomly spaced points and form deviations of type 'random only'.


Figure 2.2: Example of 2D minimum-zone circle with equidistant points and form deviations of type 'random and one harmonic'. The data set represents a three-lobbed form deviation.


Figure 2.3: Example of 2D minimum-zone circle with equidistant points. The form deviation is a superposition of a random and two harmonic systematic components.

Details for public 2D minimum-zone circle test data are listed below. These also apply to validation data sets.

## 2D Circle CO9

Number of points: 250
One random form deviation component, random position of points, full circle feature

## 2D Circle C10

Number of points: 120
One systematic harmonic and one random form deviation component, equidistant point positions, full circle feature

## 2D Circle C11

Number of points: 260
Two systematic harmonic and one random form deviation component, equidistant point positions, full circle feature

## 2D Circle C12

Number of points: 250
One random form deviation component, random position of points, full circle feature

## 2D Circle C13

Number of points: 78
One systematic harmonic and one random form deviation component, equidistant point positions, fill circle feature

## 2D Circle C14

Number of points: 216
Two systematic harmonic and one random form deviation component, equidistant point positions, full circle feature

## 2D Circle C15

Number of points: 30
One random form deviation component, random position of points, full circle feature

One systematic harmonic and one random form deviation component, equidistant point positions, full circle feature

## c) Test data sets for plane elements

Point clouds for plane elements represent coherent rectangular plane segments in 3D Cartesian space. The number of points in the PTB's different test data sets varies between 10 and 451. Some of the data sets have equidistant points placed along a rectangular grid. The list of point coordinates in the XML files are ordered in this case according to vertical and horizontal lines of the grid. The remaining plane data sets have points randomly placed on the rectangular plane segment. Here the points are not listed in a specific order in the XML files. Further all points differ from an ideal planar shape by random and systematic form deviations. Form types 'random only', 'random and convex', 'random and twisted' and 'random and saddle shaped' are implemented and are represented in different test data sets. Figures 3.1 to 3.4 give examples for point positioning and possible shape of data sets. In the figures the grey area marks the associated minimum-zone plane. Green, red and yellow dots mark measuring points. The red and yellow points have the maximum orthogonal distance from the associated plane. For better visualisation the form deviation in the examples is much higher than in the actual TraCIM test data sets.


Figure 3.1: Example of minimum-zone plane segment with form deviation of type 'random only'. The measuring points are equidistantly placed along a rectangular grid.


Figure 3.2: Example of minimum-zone plane segment with form deviation of type 'random and convex'. The measuring points are equidistantly placed along a rectangular grid.


Figure 3.3: Example of minimum-zone plane segment with form deviation of type 'random and saddle shaped'. The measuring points are equidistantly placed along a rectangular grid.


Figure 3.4: Example of minimum-zone plane segment with form deviation of type 'random and twisted. The measuring points are equidistantly placed along a rectangular grid.

Details of public plane test data are listed below. These also apply to validation data sets.

## Plane C17

## Number of points: 50

One systematic twisted and one random form deviation component, random point positions, data represents one coherent rectangular plane segment

## Plane C18

Number of points: 145
One random form deviation component, structured point positions on an equidistant $5 \times 5$ rectangular grid, data represents one coherent rectangular plane segment

## Plane C19

Number of points: 397
One systematic saddle like shaped and one random form deviation component, structured point positions on an equidistant 31x3 rectangular grid, data represents one coherent rectangular plane segment

## Plane C20

Number of points: 136
One random form deviation component, structured point positions on an equidistant $4 \times 10$ rectangular grid, data represents one coherent rectangular plane segment
Plane C21
Number of points: 10

One systematic convex and one random form deviation component, random point positions, data represents one coherent rectangular plane segment

## Plane C22

Number of points: 20
One systematic saddle like shaped and one random form deviation component, random point positions, data represents one coherent rectangular plane segment

## Plane C23

Number of points: 174
One systematic twisted and one random form deviation component, structured point positions on an equidistant $6 \times 4$ rectangular grid, data represents one coherent rectangular plane segment

## Plane C24

Number of points: 450
One random form deviation component, structured point positions on 10 parallel lines, data represents one coherent rectangular plane segment

## Plane C25

Number of points: 451
One systematic convex and one random form deviation component, structured point positions on an equidistant $11 \times 11$ rectangular grid, data represents one coherent rectangular plane segment

## Plane C26

Number of points: 35
One random from deviation component, random point positions, data represents one coherent rectangular plane segment

## d) Test data sets for sphere elements

Point clouds for sphere elements represent coherent sphere segments in 3D Cartesian space. The number of points in the PTB's different test data sets varies between 25 and 350 . Some of the data sets have equidistant points on a rectangular grid which is defined on the surface of the sphere segment. The list of point coordinates in the XML files are ordered in this case according to vertical and horizontal cross sections of the grid. The remaining sphere data sets have points randomly placed on the sphere segment. Here the point coordinate data is not in a specific order in the XML files. Further all points differ from an ideal spherical shape by random and systematic form deviation components. Form types 'random only', and 'random and one harmonic' are implemented and are represented in different test data sets.. Figures 4.1 and 4.2 give examples for point positioning and possible shape of data sets. In the figures the grey area marks the associated minimum-zone sphere. Green, red and yellow dots mark measuring points. The red and yellow points have the maximum orthogonal distance from the associated sphere. For better visualisation the form deviation in the examples is much higher than in the actual TraCIM test data sets.


Figure 4.1: Example of minimum-zone sphere with form deviation of type 'random only'. The left picture shows a data set with points placed randomly on the full sphere. The right picture shows a data set with structured positioning of measuring points on a grid with 4 vertical cross sections. These cross sections start and end on opposite sides of a diameter line of the underlying sphere.


Figure 4.2: Example of minimum-zone sphere with form deviation of type 'random and harmonic'. The left picture shows measuring points on a full sphere. The points are placed equidistantly on a grid with horizontal cross section. The right picture shows measuring data on a sphere segment. The points are placed equidistantly along vertical cross sections.

Details of public sphere test data are listed below. These also apply to validation data sets.

## Sphere C27

Number of points: 144
One random form deviation component, structured point positions on an equidistant surface grid with 5 horizontal cross sections and 10 vertical cross sections, data represents full sphere

## Sphere C28

Number of points: 350
One systematic harmonic and one random form deviation component, random point positions, data represents full sphere

## Sphere C29

Number of points: 25
One random form deviation component, point positions according to standard VDI/VDE 2617 for calibration of 3D Coordinate Measuring Machines, data represents a coherent half sphere

Sphere C30
Number of points: 182

One systematic harmonic and one random form deviation component, structured point positions on an equidistant surface grid with 6 horizontal cross sections and 8 vertical cross sections, data represents a coherent half sphere

## Sphere C31

Number of points: 50
One random form deviation component, random point positions, data represents a coherent half sphere

## Sphere C32

Number of points: 241
One systematic harmonic and one random form deviation component, structured point positions on an equidistant surface grid with 7 horizontal cross sections data represents a coherent cap of the sphere surface

## Sphere C33

## Number of points: 55

One random from deviation component, structured point positions on an equidistant surface grid with 6 vertical cross sections, data represents a coherent cap of the sphere surface

## Sphere C34

Number of points: 120
One random form deviation component, structured point positions on an equidistant surface grid with 10 vertical cross sections, data represents a coherent ring on the sphere surface

## Sphere C35

Number of points: 120
One random from deviation component, structured point positions on an equidistant surface grid with 4 horizontal cross sections, data represents a coherent ring on the sphere surface

## Sphere C36

Number of points: 210
One systematic harmonic and one random form deviation component, random point positions, data represents one coherent eighth of the sphere surface

## e) Test data sets for cylinder elements

Point clouds for cylinder elements represent coherent cylinder segments in 3D Cartesian space. The amount of points in the PTB's different test data sets varies between 30 and 480 . Some of the data sets have equidistant points on a rectangular grid which is defined on the surface of the cylinder. The grid comprises vertical and circular cross sections. These vertical cross sections are straight lines parallel to the underlying cylinder axis. Circular cross sections are curved lines orthogonal to the cylinder axis. The list of point coordinates in the XML files are ordered in this case according to the cross sections of the grid. The remaining cylinder data sets have points randomly placed on the cylinder segment. Here the point coordinate data is not in a specific order in the XML files. Further all points differ from an ideal cylindrical shape by random and systematic form deviation components. Form types 'random only', 'random and convex', 'random and concave' and 'random and conical' are implemented and are represented in different test data sets. Figures 5.1 to 5.4 give examples for point positioning and possible data set shape. In the figures the grey area marks the associated minimum-zone cylinder. Green, red and yellow dots mark measuring points. The red and yellow points have the maximum orthogonal distance from
the associated cylinder. For better visualisation the form deviation in the examples is much higher than in the actual TraCIM test data sets


Figure 5.1: Example of minimum-zone cylinder with form deviation of type 'random only'. The measuring data is equidistantly placed along 8 vertical cross sections.


Figure 5.2: Example of minimum-zone cylinder with form deviation of type 'random and convex'. The measuring data is equidistantly placed along 16 vertical cross sections.


Figure 5.3: Example of minimum-zone cylinder with form deviation of type 'random and concave'. The measuring data is equidistantly placed along $\mathbf{7}$ circular cross sections.


Figure 5.4: Example of minimum-zone cylinder with form deviation of type 'random and conical'. The measuring data is equidistantly placed along 8 vertical cross sections and 8 circular cross sections.

Details of public cylinder test data are listed below. These also apply to validation data sets.

## Cylinder C37

Number of points: 250
One systematic conical and one random form deviation component, structured point positions on an equidistant surface grid with 5 circular cross sections, data represents full cylinder feature with limited height

Number of points: 75
One random form deviation component, random point positions, data represents full cylinder feature with limited height

## Cylinder C39

Number of points: 216
One systematic convex and one random form deviation component, structured point positions on an surface grid with 5 circular cross sections and 6 vertical cross sections, data represents full cylinder feature with limited height

## Cylinder C40

Number of points: 250
One systematic concave and one random form deviation component, random point positions, data represents full cylinder feature with limited height

## Cylinder C41

Number of points: 112
One random form deviation component, structured point positions on an surface grid with 4 circular cross sections and 7 svertical cross sections, data represents $275^{\circ}$ cylinder shell with limited height

## Cylinder C42

Number of points: 264
One systematic convex and one random form deviation component, structured point positions on an surface grid with 10 circular cross sections and 6 circular cross sections, data represents full cylinder feature with limited height

## Cylinder C43

Number of points: 480
One random from deviation component, structured point positions on an surface grid with 10 circular sections and 10 vertical cross sections, data represents full cylinder feature with limited height

## Cylinder C44

Number of points: 125
One systematic conical and one random form deviation component, random point positions, data represents full cylinder feature with limited height

## Cylinder C45

Number of points: 455
One systematic concave and one random from deviation component, random point positions, data represents full cylinder feature with limited height

## Cylinder C46

Number of points: 30
One random form deviation component, random point positions, data represents full cylinder feature with limited height

## Cylinder C47

Number of points: 40
One random form deviation component, structured point positions on 4 equidistant circular cross sections, data represents full cylinder feature with limited height

## Cylinder C48

Number of points: 66

One random form deviation component, structured point positions on 6 equidistant vertical cross sections, data represents full cylinder feature with limited height

## Cylinder C49

Number of points: 50
One systematic conical and one random form deviation component, structured point positions on 5 equidistant circular cross sections, data represents full cylinder feature with limited height

## Cylinder C50

Number of points: 30
One random form deviation component, structured point positions on 3 equidistant circular cross sections, data represents full cylinder feature with limited height


[^0]:    $!\quad$ TraCIM service for testing is limited in time. After receiving an order key a customer has a total of 200 days for performing the Chebyshev test. 30 days and 3 days before the order key expires the TraCIM system will automatically send warning messages to the customer e-mail address stating the remaining time for testing. When the validity of a test expires a final information message is send to the customer.

[^1]:    Notes to table 4
    ${ }^{1)}$ Column [error code]: these codes are identical with the HTTP error codes. As one code can describe more than one reason, multiple lines with the same code are present; details can be found in the next column ([error description]).
    Column [error description]: a textual explanation of the error reason.
    ${ }^{2)}$ For this code, a 'frame message' will be generated - 'Sorry, but your request could not be interpreted. Please check your message or contact the support. The system says: '[error description]
    Please note: if, due to specific reasons, the server can not generate a proper XML error message, please check the HTML response of the server. This can be done i.e. with programming the client for processing both XML and HTML messages.

